Automatic Magnetic Control of a Momentum-Biased Observatory in Equatorial Orbit

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Theme

LOW-ALTITUDE equatorial orbits simplify high-energy astronomy by avoiding high levels of trapped particle flux. Such orbits present special problems for magnetic attitude control because torquing capability is always weak along the orbit normal. This paper shows that an observatory with a single axial wheel can be controlled accurately with a simple automatic magnetic system. The widely studied cross-product control law maintains 2-deg pointing with disturbances that would cause 2 deg/orbit open-loop drift. This law is generalized to permit a reduction in cross-axis interactions which permits 1-deg pointing. Proportional pointing laws, ignored since the early 1960's, dramatically improve pointing to the arc minute level by eliminating closed-loop interactions. (Periodic interruptions are required to correct wheel speed, possibly only during target occultation or while slewing to a new target.) The closed-loop cross-product law is modified to create a new open-loop maneuver law that uses the magnetometer alone to generate efficient slews and wheel speed corrections. The proportional pointing and maneuver laws appear to offer advantages over existing magnetic control approaches for this and other missions.

Contents

System description: A typical observatory is shown in Fig. 1. The configuration is similar to that used for SAS-C.[1] The axial gyro, momentum wheel, and star tracker are aligned with the telescope axis, here a large-area proportional x-ray counter. The gyro and wheel torque motor are used in a high-bandwidth closed loop that keeps the y axis in the equatorial plane. This restricts variations of the body-fixed magnetic field components. Like SAS-C, the system depends on ground support for initial attitude determination, slewing the telescope to the selected target and for periodic gyro update and wheel speed adjustments. The system then uses the gyro and star tracker to hold orientation automatically for extended periods.

Closed-loop control laws: A magnetic dipole moment $\bar{D}$ interacts with the geomagnetic field $\bar{B}$ to produce a control torque $\bar{T}_c = \bar{D} \times \bar{B}$. For his "magnetorquer," Kamm set $T_c = -K\Delta \bar{H}$ to remove the momentum error $\Delta \bar{H}$ and used the "efficiency condition" $\bar{D} \times \bar{B} = 0$ to avoid dipole components that produce no torque.[2] In 1961, White et al.[3] compared this three-axis "cross-product" law with continuous two-axis pointing laws that achieve "proportional" control torques $T_{cx} = -K_R \Theta_x$ and $T_{cy} = -K_D \Theta_y$, except during magnet saturation. In this paper, the efficiency condition is dropped, and generalized cross-product laws with up to six constant coefficients are compared with proportional laws. The generalized three-coefficient cross-product law gives dipole components

$$D_x = -K_S(\Delta H_z) B_y - K_R(\Theta_x H_z) B_z \tag{1a}$$

$$D_y = K_S(\Delta H_z) B_x - K_D(\Theta_y H_z) B_z \tag{1b}$$

$$D_z = K_R(\Theta_x H_z) B_x + K_D(\Theta_y H_z) B_y \tag{1c}$$

and the two-magnet proportional law gives dipole components

$$D_x = -K_R \Theta_x H_z / B_z \qquad D_y = -K_D \Theta_y H_z / B_z \tag{2}$$

where the K's are control gains, the Θ's are pointing error angles, and ΔH_z is the wheel momentum error. After linearizing and ignoring coning, the Euler moment equations for satellite motion yield momentum correction rates of

$$\begin{bmatrix} T_{cx} \\ -T_{cy} \\ T_{cz} \end{bmatrix} = \begin{bmatrix} \omega_y H_z \\ \omega_x H_z \\ \dot{H}_z \end{bmatrix} = W \begin{bmatrix} -K_D \Theta_y H_z \\ -K_R \Theta_x H_z \\ -K_S \Delta H_z \end{bmatrix} \tag{3}$$

where the ω's are body angular rates, and the "gain weighting" matrix is

$$W_C = \begin{bmatrix} (B_z^2 + B_y^2) & B_x B_y & -B_x B_z \\ B_x B_y & (B_z^2 + B_x^2) & B_y B_z \\ -B_x B_z & B_y B_z & (B_y^2 + B_x^2) \end{bmatrix} \tag{4}$$

for the generalized cross-product law, and

$$W_p = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -B_x / B_z & B_y / B_z & 0 \end{bmatrix} \tag{5}$$

for the proportional law.

Equation (4) shows that the cross-product law controls three axes simultaneously, with gains weighted by the diagonal "control field" elements, and with unwanted cross-coupling proportional to the off-diagonal "interaction fields." Without interactions, it would achieve high efficiency by driving the magnets hardest when the appropriate control field is largest. Equation (5) shows that the proportional law eliminates pointing interactions and gain variations. It applies transverse disturbances to the momentum wheel with variable multiplying factors and must drive the magnets harder to compensate for a smaller control field. Magnet saturation alters Eqs. (4) and (5) and is more likely for the proportional laws.

Open-loop maneuver law: Consider a desired maneuver (during a specified time interval) consisting of arbitrary momentum changes $\Delta\Theta_y H_z$ and $\Delta\Theta_x H_z$ (to slew the telescope) and ΔH_z (to change wheel speed). These changes can be viewed as integrals of the left-hand side of Eq. (3). Equation (4) can be integrated in a gound computer (with an appropriate field model) to determine the average fields. Three "maneuver constants" can be determined by premultiplying the integrated version of Eq. (3) by the inverse of the matrix of average fields. If these constants are substituted for the coefficients of the field components in Eq. (1), then the magnetometer outputs alone will program the dipole

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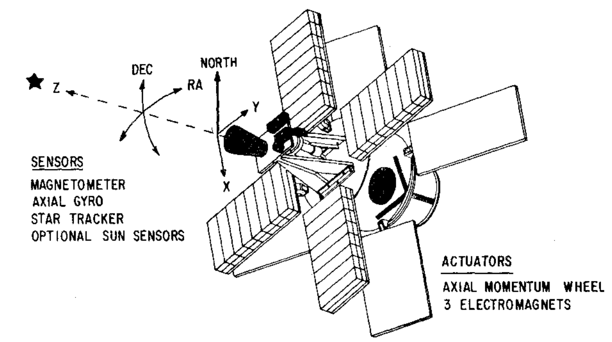

Fig. 1 Typical satellite configuration.

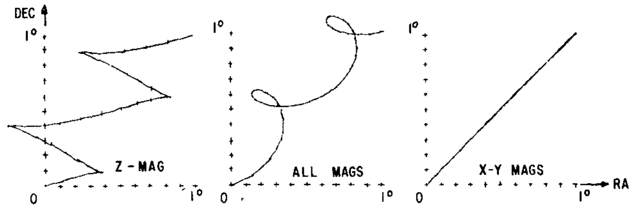

Fig. 2 Effect of magnet choice on slews (TDEC = 45°).

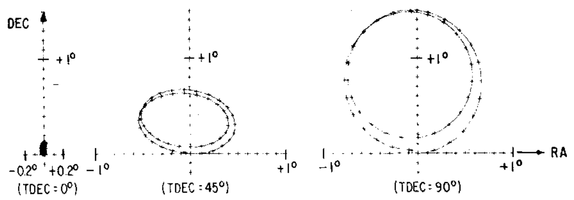

Fig. 3 Pointing perturbation during wheel speed change (all magnets).

moments to approximate the desired maneuver. To illustrate the maneuver law, we apply it first to slew the telescope and next to change wheel speed. All results are from a digital computer simulation using a tipped-centered geomagnetic dipole model.

Open-loop slew maneuver: Figure 2 shows how magnet choice effects the slew path of the telescope for a one-orbit 1-deg slew in both RA and DEC starting at mid-declination. Path irregularities are caused by orbital variations of both the control and interaction fields. The left path using only the Z magnet is most erratic and requires the largest dipoles but does not change wheel speed. The large dipole that must be used to compensate for a small DEC control field magnifies the effect of the RA-DEC interaction field. For transverse magnets only, the slew path is straight, but the slew rate is irregular and there is a secular wheel speed change. The all-magnet slew is most efficient and eliminates the secular speed change.

Open-loop wheel speed change maneuver: Figure 3 shows the telescope axis perturbation during commanded one-orbit open-loop changes of 5% of the nominal wheel momentum. The three maneuver constants were calculated to return the telescope to the origin. For all three declinations, the peak perturbation is small compared to the 8-deg field-of-view of the star tracker. The perturbation is largest at high declination because both transverse magnets need large dipoles to compensate for the small equatorial components of the field. The DEC perturbation is biased because there is a bias in the SPIN-DEC interaction field.

Cross-product law performance: The basic concept of the cross-product law is to rely on field direction changes to extend planar control to three axes. To implement this concept, the control bandwidth must be low enough to "blur" the plane normal to $\bar{B}$. This is equivalent to absorbing the disturbance along $\bar{B}$ for large fractions of an orbit. Orbital changes in the control and interaction fields cause cyclical

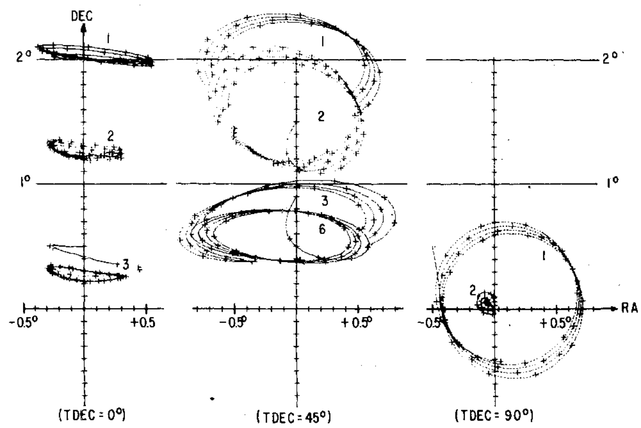

Fig. 4 Cross-product law pointing improvement.

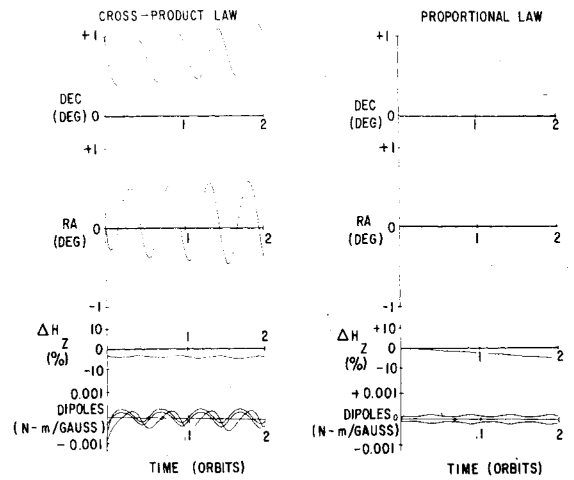

Fig. 5 Pointing comparison (TDEC = 45°).

error variations. Figure 4 shows how the additional coefficients in the generalized cross-product law may be optimized to reduce the interactions. Each set of disks represents two orbits of uninterrupted pointing with typical bias disturbances. The numbers indicate how many different coefficients are used. Errors are largest for mid-declination because of the bias in the SPIN-DEC interacting field. This is the principal cross-product law performance limiter.

Proportional pointing law performance: The basic concept of the proportional law is to generate desired transverse control torques independent of the misalignment between this plane and the magnetic torque plane (normal to $\bar{B}$). For a mid-declination target and typical bias disturbance torques, Fig. 5 shows the dramatic pointing improvement compared to the generalized cross-product law. The figure compares two-orbit time histories of pointing error angles, wheel speed, and magnet usage. The proportional gains were set to maintain static errors smaller than 1 arc min. Wheel speed is not controlled by the proportional laws, and periodic adjustment would be necessary. For low targets, the proportional laws periodically saturate the magnets and allow brief pointing excursions beyond 1 arc min (4 arc min was the largest simulated). For slightly inclined orbits, the errors vary diurnally, and the worst case for any magnetic controller is for an orbit inclined at the tip-angle of the geomagnetic field.

References

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[3] White, J. S., Shigemoto, F. H., and Bourquin, K., "Satellite Attitude Control Utilizing the Earth's Magnetic Field," NASA TN-D1068, Aug. 1961.